Number Theory Class 1 - Problems

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No calculators!

1 Review problems

1. Jon teaches a fourth grade class at an elementary school where class sizes are always at least 20 students and at most 28. One day Jon decides that he wants to arrange the students in their desks in a rectangular grid with no gaps. Unfortunately for Jon, he discovers that doing so could only result in one straight line of desks. How many students does Jon have in his class?

2. Small composites

   a. Find the smallest composite number not divisible by 2.

   b. Find the smallest composite number not divisible by 2 or 3.

   c. Find the smallest composite number not divisible by 2, 3, or 5.

   d. For $p \in \mathbb{P}$, find the smallest composite with no prime divisor less than p.

3. One number theory textbook I've found "defines" the composite numbers as the set complement of $\mathbb{P}$ with respect to $\mathbb{N}$ - the natural numbers that aren't prime. Why is this wrong?

4. Find the GCD of...

   a. 990 and 720

   b. 819 and 504

c. 2001 and 25001

d. The first n primes ($n \in \mathbb{N}$)

5. Find the LCM of...

a. 18 and 42

b. 21 and 63

c. 135 and 144

d. The first n primes ($n \in \mathbb{N}$)

6. Find a relationship among the prime factorizations of all positive multiples of 45.

7. What is the greatest prime divisor of $1 + 2 + 3 + \ldots + 70$?

8. The larger of a pair of integers is 8 times the smaller. Why can't they both be perfect squares?

9. The smaller of a pair of integers is 8 times the larger. Why can't either be a perfect square?

10. Find GCD(347188444,231458962). Hint: the prime factorizations are nasty.

2 Challenge problems

1. Let the prime factorization of a number be $p_1^{a_1} p_2^{a_2} \ldots p_n^{a_n}$. How many distinct divisors does it have?

2. What is the smallest prime factor of $5^{23} + 7^{17}$?

3. Suppose that a, b, m, and n are natural numbers. Explain why $GCD(m, n) = GCD(m, am + bn) = GCD(am + bn, n)$.